STEFAN WOLFART

IMPLANT PROSTHODONTICS

A PATIENT-ORIENTED CONCEPT

PLANNING | TREATMENT PROCEDURES | LONGEVITY | ESTHETICS | FUNCTION | DENTAL TECHNOLOGY

WITH CONTRIBUTIONS BY
SÖNKE HARDER, SVEN REICH, IRENA SAILER, VOLKER WEBER

QUINTESSENCE PUBLISHING

Berlin, Chicago, Tokyo, Barcelona, Istanbul, London, Milan, Moscow, New Delhi, Paris, Prague, São Paulo, Seoul, Singapore and Warsaw

Fig 3-29 Baseline situation with an irregular gingival line, recessions around teeth 22 and 23 (Miller Class I); restoration is indicated for teeth 11 to 22.

Fig 3-30 Appearance following a coronally advanced flap and connective tissue graft around teeth 22 and 23, in-office bleaching of all the teeth, and adhesive retention of the all-ceramic restorations of teeth 11, 21, and 22.

The case example shows horizontal widening of the pontic bed with a connective tissue transplant (Figs 3-31 to 3-34). Following a 3-month graft-healing period, the concave shape of the pontic bed was premodeled with a coarse round diamond bur (Fig 3-32) and then shaped with the provisional restoration (Fig 3-33). The Fit Checker test on the restoration shows that the ovate pontic fits this molded shape exactly (Fig 3-34).

3.2.7 Category nine: neighboring teeth

To achieve a successful esthetic appearance overall, it goes without saying that the planning should include not only the teeth due for restoration but also their neighbors. It is often necessary to make changes in the color, size, shape, or position of the neighboring teeth to achieve a harmonious overall result. Whether these changes are to be accomplished with bleaching methods, restoration measures or orthodontic aids will need to be decided on a case-by-case basis (see Fig 3-24 versus Fig 3-27).

3.2.8 Category ten: smile line

The line of the lower margin of the upper lip acts as a guide to the visibility of the teeth. The extent to which the upper lip's lower margin is raised when a person speaks and smiles, and by how much this exposes the anterior teeth and the gingiva of the maxilla, is used to classify the smile line into three visual-effect categories[313]:

1. Incisal effect: the incisal third to incisal half of the maxillary teeth is visible.
2. Cervical effect: the dental arch is visible up to the tips of the papillae.
3. Gingival effect: larger sections of the maxillary gingiva are exposed along with the teeth (gummy smile).

A very pronounced cervical effect and, in particular, the gingival effect, can negatively affect the esthetics of the smile if there are any imperfections in the restorations and the neighboring soft tissue. In implantology, these imperfections include exposed implants, abutment components, and visible junctions between the acrylics or pink ceramic of prostheses and the oral mucosa. In classic prosthodontics, they include visible crown margins and the pontics of partial dentures, which impair the esthetic result. These problems are aggravated as soon as any more extensive vertical and/or horizontal soft tissue defects develop.

An anterior tooth implant that is too wide and has been placed too deep, with its shoulder exposed, is one example of a situation in which the smile line decides the esthetic success or failure of the treatment (Fig 3-35). Since soft tissue coverage of the defect has little chance of success, the alternative to explantation is renewed, compromise-ridden loading of the implant. In the case shown here, this was done by grinding the implant and abutment to the required shape (Fig 3-36) and then loading them with a metal-ceramic crown (Fig 3-37). When the restoration was incorporated, the junction between the pink ceramic and the natural gingiva leapt to the

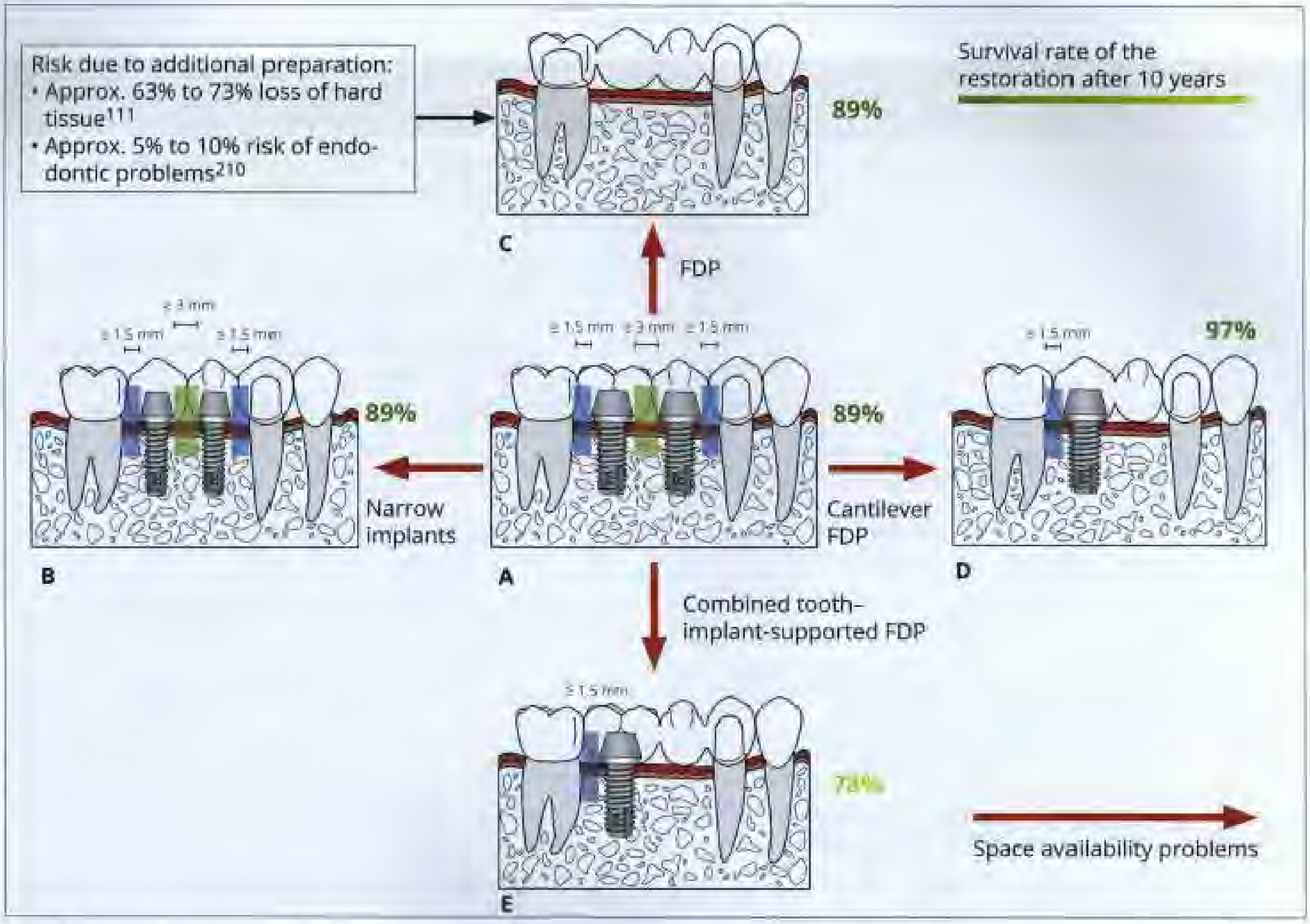


Fig 4-1 To avoid bone loss between two implants or between tooth and implant, the distance between implants should be at least 3 mm and the distance between tooth and implant at least 1.5 mm *(A)*. If these distances cannot be maintained due to space, four alternatives are available for multi-tooth gaps, with different restoration survival rates: *(B)* the use of narrow implants; *(C)* preparation of a conventional end-to-end FDP with the corresponding disadvantages for the abutment teeth; *(D)* an implant-supported cantilever FDP; *(E)* a combined tooth–implant-supported FDP, though this has the worst 10-year prognosis. An implant-supported FDP is another suitable option for larger gaps of three or more missing teeth.

Not least, the *suitable number of implants* depends on the available space: to preserve the approximal bone and the papilla, the minimum distance between implant and tooth should be 1.5 mm, and the minimum distance between two implants should be 3 mm[369]. If it is not possible to maintain these distances, it may make sense to leave out one implant and to work with an implant-supported FDP (end-to-end or cantilever FDP) or a combined tooth–implant-supported FDP (Fig 4-1).

For the same reason, a reduced implant diameter is also required when replacing mandibular incisors (mesiodistal diameter approximately 5 to 5.5 mm) and, in some cases, also maxillary lateral incisors (mesiodistal diameter approximately 6.5 mm). Since the mechanical, load-bearing capacity of narrow-platform implants is considerably reduced, their use should be restricted to the aforementioned regions where space is limited and, where occlusal load is likely to be relatively low.

Table 4-3 gives an overview of the restoration concepts for the single-tooth gap, the multi-tooth gap, and the free-end situation.

Fig 7-6 Constructing a provisional restoration on the abutment with the aid of a vacuum-formed template (the screw access hole has been blocked off with blue wax).

Fig 7-7 Due to the cylindrical emergence profile, only the coronal segment of the provisional restoration is the correct shape (*dashed line* shows the desired profile).

Fig 7-8 Acrylic resin has been used to correct the shape of the emergence profile and the screw access hole has been exposed.

Fig 7-9 The provisional restoration has been put into place, causing initial ischemia of the mucosa *(arrow)*.

Fig 7-10 Shaped emergence profile.

Fig 7-11 Definitive all-ceramic restoration *(lithium disilicate glass-ceramic crown)* with individualized *zirconia abutment*. The abutment complements the sweep of the crown and leads it back smoothly to the circular abutment shoulder.

Complications with implant-supported restorations can have serious consequences for the overall course of an implant treatment and for our patients' stomatognathic systems. These complications generally occur only after a few years. For this reason, review papers and meta-analyses on dental prostheses usually specify a minimum follow-up period of 5 years as an important inclusion criterion. A prosthesis can be considered to have long-term success after this period of time. However, 5-year survival data are not available for a few types of restorations that we use in routine daily practice.

Given the dearth of adequate scientific data and the consequent lack of standardized treatment concepts in routine practice, similar clinical situations may be addressed with different numbers of implants, as well as different implant/abutment joints and retention elements. The resultant multitude of treatment concepts for each type of gap in the dentition makes it more difficult to perform a systematic analysis of the complications that frequently occur in implant prosthodontics.

This chapter distinguishes between complications arising because the patient has been given inadequate information, and complications associated with fixed and removable restorations. Sections 20.1 and 20.2 are largely taken from an article by Wolfart et al[411].

20.1 Complications arising from inadequate patient information

In view of the wide and uncertain treatment corridor available to us, it is essential to involve patients actively in the decision-making process as regards the form of treatment that best fits their specific circumstances[328]. This raises the possibility of inadequate patient information as the first significant potential source of complication, which also results in inappropriate treatment planning for the patient. This type of poor treatment decision can lead to many years of compromise regarding the patient's oral health-related quality of life. When, after many years, the dental prosthesis is finally replaced with a new restoration to correct the previous unsuitable choice of treatment, it often prompts patients to ask why they did not have the treatment before, or how they coped with their old denture for so many years.

Comprehensive, detailed consultations with the patient are needed to avoid these potential years of dissatisfaction. To do this, the dentist needs to possess extensive knowledge of the various forms of restoration available and their effect on oral health-related quality of life. The importance of this issue was demonstrated in a study concerning the rehabilitation of the edentulous maxilla with fixed or removable restorations[171].

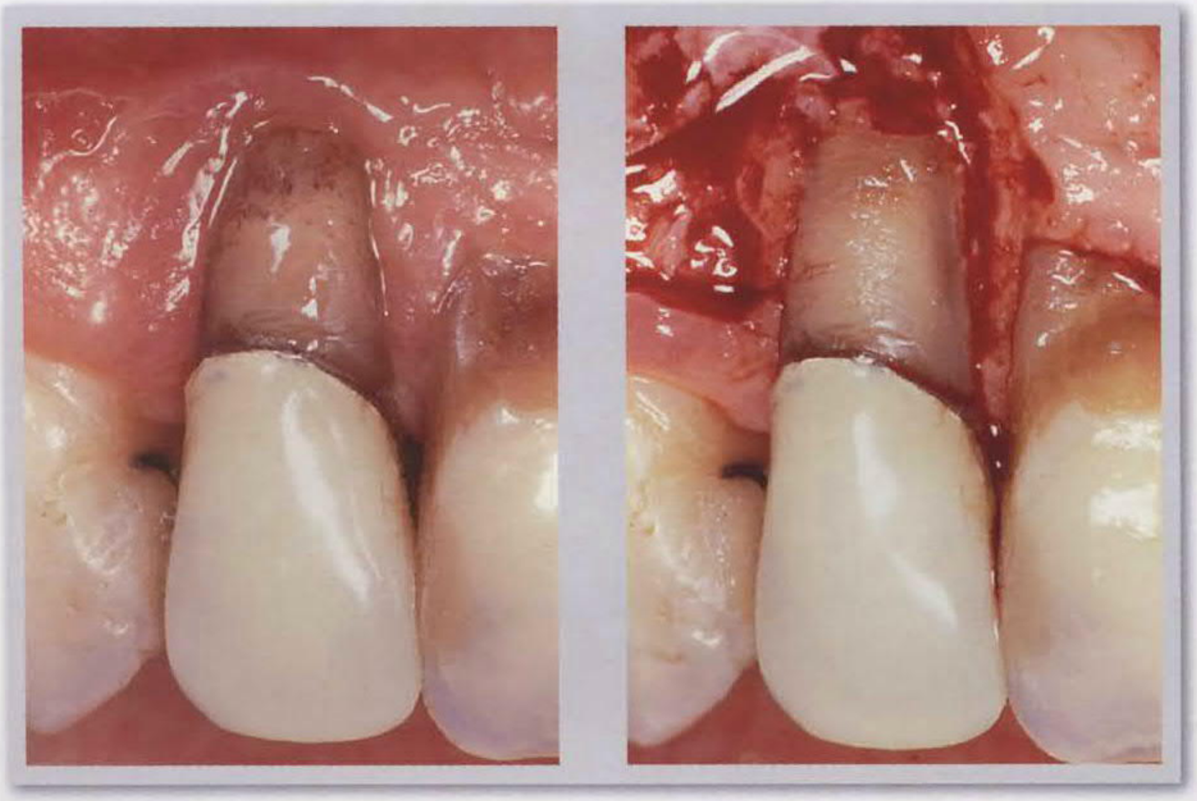
"Many unhappy treatment courses and complications result from inadequate patient information. This is rarely intentional, but is often due to an incomplete understanding of the patient information process:

"'Informing the patient' does not mean merely conveying information about a treatment plan (as recommended by the dentist), but also includes both full '*procedural information*' and detailed '*safeguarding information*'[141,297,318]. The *procedural information* must precede any measure that encroaches on the prosthodontic patient's physical integrity; it should convey to the patient a general idea of the nature and severity of his or her disorder and of the potential implant treatment (urgency, chances of success, scope, treatment alternatives, risks and side effects, likely further action in the event of a treatment failure), thus putting the patient into a position of being able to make his or her own informed decision about the recommended measures. This is the only way dentist and patient can embark on shared decision-making. Conversely, the purpose of the *safeguarding information* is to inform the patient about the do's and don'ts related to the treatment: thus, for example, this information process might start after the implants have been inserted, and aims to assure successful healing by reinforcing the patient's own responsibility for the period after the implant-supported prosthetic restoration has been placed. This includes warnings about the possible consequences of unhealthy behavior and potentially damaging habits, as well as recommendations on concrete changes in lifestyle which are of key importance to the healing process (eg, changes in oral hygiene measures and dietary habits which are relevant to the patient's individual circumstances).

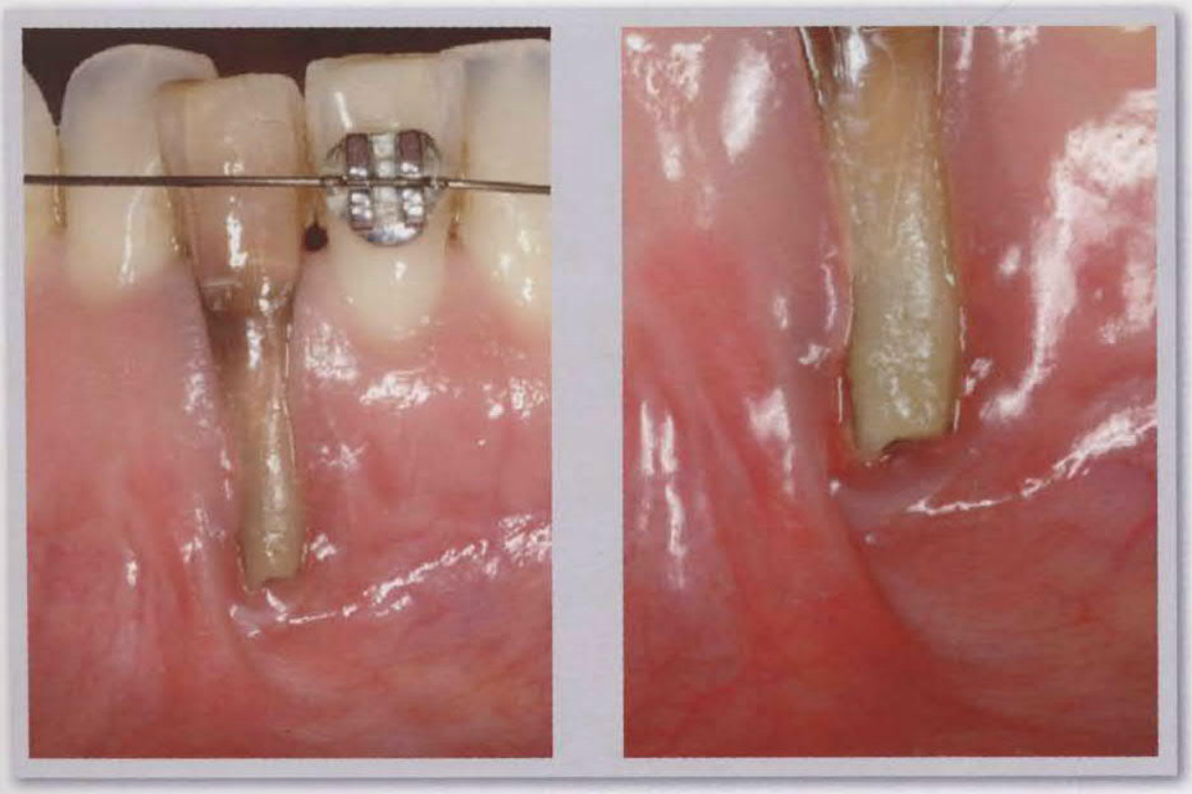
"One of the dentist's fundamental duties during every information talk is to ensure, by means of *repeated questioning*, that the patient has 'taken in' and understood the information that was provided; it is also just as important to signal to the patient that it is absolutely appropriate for him or her to decide for or against the planned implant-supported restoration only after taking *sufficient time to think it over*.

"Consequently, providing full and consistent patient information is not just a legal and ethical requirement, but also the first cornerstone on the way to a successful treatment: it increases the patient's understanding of the existing treatment situation, provides concrete options

Areas of gingival recession may also form as a result of an inflammatory reaction coming from periapical tissue infection (plaque-induced apical gingival recession). The cause may be reinfection following endodontic surgery.



Or the cause may be untreated previous periapical infection.



Among the various root coverage techniques, the coronally advanced flap is the method of choice in that it is a straightforward procedure, is well tolerated by the patient (ie, good postoperative recovery), gives excellent results from an esthetic point of view because of its high percentage of complete root coverage, and camouflages the area treated to blend well with the adjacent soft tissues. The author believes that an esthetic outcome must always be aimed for even if not explicitly requested by the patient.

The keratinized tissue apical to the exposed root is considered adequate for a coronally advanced flap (see chapter 17) when:

- Its thickness and apicocoronal dimension are suitable (according to the patient's biotype and the depth of gingival recession)
- The vestibule is sufficiently deep
- There is no deep cervical abrasion
- The tooth root is not displaced too facially

When the keratinized tissue is not adequate for a coronally advanced flap, first the patient's expectations regarding esthetics must be investigated, including his or her feelings regarding the possibility of a less than ideal outcome. If the patient's esthetic demands are high, evaluation should be made to determine if a minimal residual amount of keratinized tissue exists apical to the exposed root area. If so, the technique to use is a coronally advanced flap covering a connective tissue graft placed apical to the CEJ (see chapter 22). Where there is no remaining keratinized tissue apical to the exposed root area, assessment should be made of the keratinized tissue distal to the root exposure. If this is suitable, the treatment indicated is a coronally advanced lateral sliding flap (see chapter 18). If there is inadequate keratinized tissue quantity and quality distal to the recession, it should be evaluated whether there is traction from muscles or frenula inserting into the gingival margin and whether the vestibule is shallow.

A sufficiently deep vestibule and lack of marginal frenula indicate suitability of a bilaminar technique consisting of a coronally advanced flap covering a connective tissue graft placed over the CEJ (see chapter 22). The presence of marginal frenula or a shallow vestibule make it necessary to perform a two-step technique. The depth of the bone dehiscence determines the choice between a free gingival graft placed apical or lateral to the gingival recession (see chapter 20).

If the patient has no esthetic requirements or demands, the technique of choice (after the coronally advanced flap) is the lateral sliding flap. In this case the keratinized tissue sufficient to perform the procedure can be found either mesial or distal to the recession. The absence of a palatal harvest makes the lateral sliding flap better tolerated by the patient when compared with a coronally advanced flap over a connective tissue graft placed apical to the CEJ (which, however, permits better esthetic results). If the keratinized tissue lateral to the recession is not adequate for a lateral sliding flap it is necessary to evaluate whether there is a minimal residual height of keratinized tissue apical to the exposed root. Should this be the case, the technique to choose is the coronally advanced flap covering a connective tissue graft placed apical to the CEJ (see chapter 22). Conversely, a total absence of apical keratinized tissue makes it necessary to evaluate whether there is traction from muscles or frenula inserting into the gingival margin or whether the vestibule is shallow. A sufficiently deep vestibule and lack of frenula indicate a bilaminar technique consisting of a coronally advanced flap covering a connective tissue graft placed over the CEJ (see chapter 22).

The presence of marginal frenula or a shallow vestibule make it necessary to assess the width of the recession defect. Root coverage of a narrow, shallow defect is obtained with a free gingival graft (see chapter 19). A wide defect instead requires a two-step technique. The depth of the bone dehiscence determines the choice between a free gingival graft placed apical or lateral to the gingival recession (see chapter 20).

By preventing correct oral hygiene, imprecise dentures (eg, poorly constructed provisional restorations or overhanging margins) may be a predisposing factor to the onset of gingival recession. Traumatic lesions from brushing or flossing (similar to gingival clefts) may form, or gingival recession may follow localized plaque accumulation.

Where gingival recession is caused by traumatic interproximal attachment loss during abutment tooth reduction, root coverage cannot be obtained with mucogingival surgery. On the contrary, if the denture is merely a factor predisposing recession, root coverage is feasible. In both cases, new properly fitting dentures should be fabricated. Where interproximal attachment loss has occurred, the facial margin of the new dentures must be apical to that of the previous dentures.

4

Specialized Techniques
LOW-FIRE ELECTRIC REDS

by David L. Gamble

Above: Plate, by David Gamble. Cross is glazed with red underglaze.

Left: Untitled, by Scott Bennett. Amaco LM series Coral glaze with wax and Black overspray. As the wax melts in the kiln, the black moves.

I'll start by explaining there are two different types of commercial red glazes that I normally use. One type is an extremely bright color and harder to achieve and the other is a newer tomato red color that is AP (Approved Product of the Arts and Creative Materials Institute) nontoxic and dinnerware safe. The latter is formulated with inclusion stains, which are continuing to be improved. The color is encased in zircon, which makes them safe to use even in the classroom.

The AP nontoxic reds are extremely stable and were used to create red velvet underglazes that can be fired from cone 05 to as high as cone 10—only salt seems to blush them out.

The success of underglazes has allowed the development of gloss and matt red glazes that have been formulated to work well at the low-fire cone 05 range and other glazes formulated for the cone 4–6 range. These are extremely reliable. Three brushed coats will usually be enough of an application and you get nice tomato color reds at both temperatures.

Bright reds are not dinnerware safe and are extremely sensitive to variations in firing conditions. There have been many times that an art teacher has asked me about the use of these types of red glazes. I understand the space and time challenges that teachers face, but you cannot put these glazes in with your normal glaze firings and expect good results. They are affected by how tight the load is stacked, other glazes (mostly copper greens), and temperature. If you're firing to cone 05, I can almost guarantee there will be problems. The glaze will most likely have variations from clear to gray to black, and if you're lucky, a spot or two of red. Note: Amaco glazes were used in the pieces shown here, however, many companies produce similar glazes.

Redhot Chilli Pepper Diner, by Jerry Berta. Glazed with red underglazes.

Process

Here are my suggestions of what you need to know and do to achieve the bright reds.

- Bisque your clay body slowly to cone 04 (12 hours to get all the gases out). Although these glazes are not considered translucent, the clay body color does affect them slightly. White bodies will make the glaze appear brighter in color than darker bodies.
- Using a brush, apply the glaze thicker than the normal three coats. Four coats will usually work, but too heavy an application may cause the glaze to run. Glaze application may need experimentation and practice.
- Load the kiln very loosely. There is a need for lots of space between the pieces for air circulation. I leave the peephole plugs out during the firing, thus allowing extra oxygen to enter the kiln chamber.
- Do not fire above cone 06 (1828°F), preferably using witness cones for observation. I have been firing at cone 07 (1789°F) with great results. These glazes seem to like the cooler temperatures.
- Fire as quickly as you can, four hours is ideal. If your pieces are larger, an example being my 22-inch platters, take them up slowly to about 1200°F. This may help to eliminate cracking problems. Then turn the kiln on high to fast fire to the end of the firing.

Observations

If your kiln is vented through the bottom with a system that draws air through the top of the kiln, this will help give you more oxygen in the kiln and better red results. Remember that kilns, depending on how they are stacked, may not fire that evenly. This can cause cold spots and hot spots. There can be a difference in temperature equal to a couple of cones from top to bottom—depending where the kiln sitter or thermocouple is located. This variability can really affect bright red glazes. Newer kilns with zone control and multiple thermocouples tend to fire more evenly. If you have an older kiln, place cones in the top, middle and bottom of the kiln so you can keep a record of what happens in the firing. They can help provide answers if problems do occur.

I've been placing red glazes on different color clay bodies, layering over glazed fired pieces and layering one coat of gold glaze over the top.

I then place the pieces next to peep holes to brighten the color, or place shelves over the edges to deepen and take away the color. This is what is exciting to me—not getting it perfect, but having the surface color change and vary while having some control over what the changes will be. I am an avid advocate of using commercial glazes the way a painter would use his tubes of paint. Experiment, test to the "max" and make them your own. Don't be afraid to experiment.

Platter detail, by David Gamble, glazed with red glaze, blue brush strokes and one coat of gold used for accents.

Platter, by David Gamble, glazed with red glaze and blue brush strokes on top.

Specialized Techniques

MASTERING MICA

by Kate and Will Jacobson

For the past year, we've been exploring the subtle luster and compelling color palette of mica as a glaze element. We usually teach naked raku, but wanted to give our students some other low-fire techniques to explore. While preparing for a workshop, we tested the reaction of various terra sigillatas, colored porcelain slips, and even acrylic paint in the ferric chloride saggar process.

Why did we try a copper-colored acrylic paint? We discovered the pigment in the copper color is mica coated with titanium and iron. We thought it would be a good source for these oxides. Turns out, it was a good source of mica.

Mica, a mineral often used in cosmetics for it shimmery essence and in electronics for its insulating properties, is a very refractory mineral. It easily withstands the 1472°F (800°C), (cone 015), temperature a lot of bare-clay firing techniques call for, making it ideal for using in several low-fire techniques such as naked raku, ferric chloride saggar, horsehair firing, clay saggar and pit-fired ceramics. Detailed explanations of these firing techniques are well covered in the book *Naked Raku and Related Bare Clay Techniques* published by The American Ceramic Society (www.CeramicArtsDaily.org/bookstore).

Glazing with Mica

There are several ways to use mica as a glaze element in low-fire techniques. Wearing a dusk mask, mix 5 grams of mica powder into one cup of terra sigillata made from OM4 ball clay.

Life Aquatic, sponged and painted with copper colored "mica paint" and block printed on the surface, ferric-chloride fired.

This will give you a starting point for your color. The more mica you add, the more saturated the color becomes.

Next, brush two coats of plain terra sigillata on a bone dry piece. Then apply a topcoat of the mica sigillata. This can be brushed, sponged, painted, stamped, sprayed, etc. (figures 1 and 2).

1. Applying mica paint (matte acrylic medium mixed with ½ gram of colored mica powder) with a stamp.

2. Brushing a top coat of Jacobson's Super Copper mica sigillata on top of 2 coats of regular OM4 sigillata.

3. *Wave*, OM4 sigillata, copper-colored mica paint sponged on surface, ferric chloride fired.

4. Applying colored mica dry rub onto a slightly dry, textured clay surface with a soft bristle make-up brush.

When dry, burnish with a piece of plastic wrap and bisque fire to 1382°F, (cone 017). Your piece is now ready to be used in one of many low-fire techniques. The mica gives an added luster and subtle sheen that emanates from within the clay. This application also works well with any bare clay technique that fire at or under 1472°F.

Making Mica Paint

Another way to use mica is to mix your own mica paint. This is particularly effective in ferric chloride, aluminum-foil saggar firings. Mix two tablespoons of matte acrylic medium (available at art supply stores) with ½ gram of colored mica powder. Paint or sponge this mixture onto an already bisque-fired pot that has been coated with either regular OM4 terra sigillata or mica sigillata.

Once dry, and wearing latex gloves and a respirator, paint or pour ferric chloride on the piece. Then, wrap the piece with two layers of aluminum foil, making sure you get a tight seal. Fire the piece rapidly to 1472°F, then back off the temperature to 1382°F and hold for 10 minutes. Warning: Wear an appropriate respirator when firing with ferric chloride as you must take extreme caution to not inhale the fumes.

After firing and unwrapping the piece, take a soft brush and remove some of the residual dust.

5. *Pineapple,* 10 inches (25 cm) in height, Laguna Amador Clay, mica dry rub, heated and smoked.

In order to fix the surface, use a UV-resistant fixative spray to seal and protect (figure 3).

We have discovered that one of the properties of mica is that it does not trap carbon. This is good news because it allows for contrast between the clay and mica in a smoke firing. This technique works well with highly textured forms. We like to call this the 'dry rub' technique. Use a soft bristle make-up brush to scrub dry mica powder onto a not-quite-leather-hard pot (figure 4). The mica is pushed into the clay and then the excess is brushed off. Bisque fire the piece to cone 017. Now it's ready to fire in a raku kiln followed by reduction in a smoke chamber. The result is shimmering mica embedded into the clay juxtaposed against the matte black of the carbon-infused clay (figure 5).

Mica Sources

- www.TKBTrading.com: TKB Trading has hundreds of colors to choose from. We have tried about 50. The reds seem to change or fade. The greens hold up nicely. The blues change a little but hold up okay. The copper, gold, and the silver colors hold up the best. The ferric chloride saggar technique is the hardest on the mica and causes more fading and color change. The colors we recommend are Breath of Spring, Deep Blue, Pearl Green, Emerald, Pennsylvania Green, Swiss Chocolate, Patagonia Purple, Glitter Siena, and Gold Lamé.
- www.EarthPigment.com: From Earth Pigments, we recommend Super Copper and Sterling Silver.

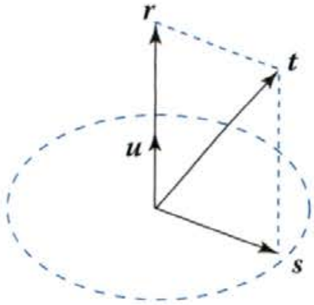


Fig. 2.19 Plane projection.

The challenge is to find the projection tensor N that projects vector t onto the plane given by u. The projected vector is labeled s. From the vector triangle we derive

$$s = t - r$$

and substituting Eq. (2.21) for r

$$s = t - Pt = (E - P)t = (E - u\bar{u})t$$

We define the plane projection tensor with E the unit tensor and u normal of the plane

$$N = E - u\bar{u} \tag{2.25}$$

Just like the line projection tensor, the plane projection tensor is symmetric.

Example 2.8 Focal Plane Imaging

Problem:

An aircraft is imaged on a focal plane array. To simulate that process, we need to develop the equations that project the aircraft's silhouette on the focal plane. We keep it simple by modeling the perspective of the aircraft with the displacement vectors of the tip, stern, right wing tip, and left wing tip wrt the geometrical center C, t_{B_1C}, t_{B_2C}, t_{B_3C}, and t_{B_4C}. The displacement of the aircraft center C wrt the focal plane center F is given by t_{CF} and the orientation of the planar array by the unit normal vector u. Separation distance and optics reduce the scale of the projections on the focal plane by a factor f. Determine the aircraft attitude vectors s_{B_1C}, s_{B_2C}, s_{B_3C}, and s_{B_4C} and the displacement vector s_{CF} in the focal plane. (To practice, make a sketch.)

(Continued)

Example 2.8 Focal Plane Imaging (*Continued*)

Solution:

Subjecting the displacement vectors to the plane projection tensor $N = E - u\bar{u}$ and reducing the magnitude by f produces the image

$$s_{B_1C} = fNt_{B_1C}, \quad s_{B_2C} = fNt_{B_2C}, \quad s_{B_3C} = fNt_{B_3C}, \quad s_{B_4C} = fNt_{B_4C}$$

and the displacement of the aircraft from the focal plane center

$$s_{CF} = fNt_{CF}$$

For building the simulation, the vectors have to be converted to matrices. Most likely, the aircraft data are in geographic coordinates $]^G$, and the image should be portrayed in focal plane coordinates $]^F$. Therefore, a transformation between the two coordinate systems $[T]^{GF}$ will enter the formulation.

2.3.6 Reflection Tensor

A plane of symmetry has the characteristics of a mirror. The left side repeats the right side. Any aircraft exhibits this reflectional symmetry—whatever happened to the oblique wing? Even right-hand maneuvers can be reflected into left-hand maneuvers just by geometrical manipulation. The tensor that makes this happen is called the reflection tensor M.

What are the characteristics of this tensor M that reflects the vector t into t' by the mirror plane with unit vector u (see Fig. 2.20)? First, project t onto u with the projection tensor $P = u\bar{u}$ to get $r = Pt$ and then derive t'

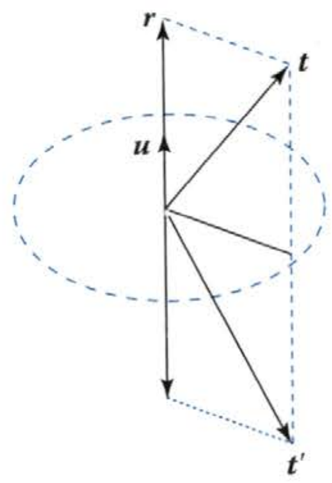


Fig. 2.20 Reflection.

from the vector triangle

$$t' = t - 2r = t - 2Pt = (E - 2P)t$$

The reflection tensor of the mirror plane with unit normal u and unit tensor E is therefore

$$M = E - 2u\bar{u} \tag{2.26}$$

It is not only symmetric but also orthogonal, as we can demonstrate by

$$M\bar{M} = (E - 2u\bar{u})(E - 2u\bar{u}) = E - 4u\bar{u} + 4u\bar{u}\,u\bar{u} = E$$

The reflection tensor plays an important role in Chapter 7, where we will sort out the existence of higher-order aerodynamic derivatives of airframes exhibiting reflectional symmetry. If the mirror plane is oriented in body coordinates $]^B$ such that its unit normal has the coordinates $[\bar{u}]^B = [0 \quad 1 \quad 0]$ (right wing of aircraft), then the reflection tensor has the coordinates

$$[M]^B = [E]^B - 2[u]^B[\bar{u}]^B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We conclude that the reflection tensor changes the sign of the second coordinate, but keeps the other two coordinates unchanged.

Example 2.9 Application of Reflection Tensor

Problem:

An aircraft (see Fig. 2.21), with a canted twin tail, executes a pushdown maneuver. What is the resultant force of both control surfaces f if the force on one surface is f_1? Derive the equations in invariant form and then introduce body coordinates $]^B$.

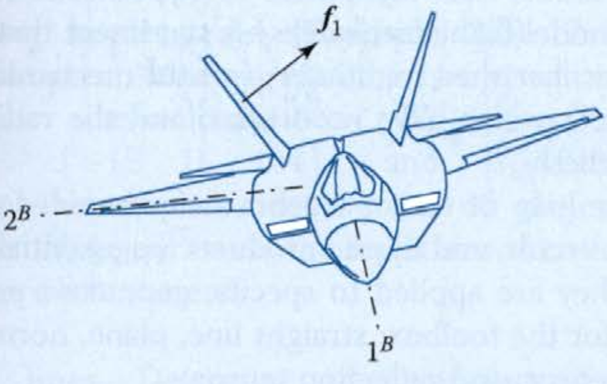


Fig. 2.21 Control forces.

(*Continued*)

Example 2.9 Application of Reflection Tensor (*Continued*)

Solution:

We use the reflection tensor to determine the force on the other control surface and add both together

$$\boldsymbol{f} = \boldsymbol{f}_1 + \boldsymbol{f}_2 = \boldsymbol{f}_1 + \boldsymbol{M}\boldsymbol{f}_1 = (\boldsymbol{E} + \boldsymbol{M})\boldsymbol{f}_1 = 2(\boldsymbol{E} - \boldsymbol{P})\boldsymbol{f}_1$$

With the unit normal of the mirror plane being $\boldsymbol{u}$

$$\boldsymbol{f} = 2(\boldsymbol{E} - \boldsymbol{u}\overline{\boldsymbol{u}})\boldsymbol{f}_1$$

This is the desired result in symbolic tensor notation. Introducing body coordinates for the unit normal $[\overline{u}]^B = [0 \quad 1 \quad 0]$, and for the force on the right control surface $[\overline{f}_1]^B = [0 \quad f_{12} \quad f_{13}]$ yields the result

$$[f]^B = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ f_{12} \\ f_{13} \end{bmatrix} = 2\begin{bmatrix} 0 \\ 0 \\ f_{13} \end{bmatrix}$$

We find that the horizontal force components cancel, and the vertical component is doubled by the second surface.

2.4 Summary

If you are reading this, you have persevered until this chapter's end. We are indebted to the physicists of the 18th and 19th centuries for the foundations of classical mechanics. Its axiomatic treatment puts our modeling tasks on a sure footing. Simple Cartesian tensors in Euclidean three-space are the symbolic language, and their realization as matrices by coordinate systems are the fodder for computers. I hypothesized that points and frames suffice to model flight mechanics—a statement that still needs verification. Some of the cherished traditions of vector mechanics had to be abandoned. Coordinate systems have no origins, and the radius vector has no place in our tool chest.

Then, with the help of tensor algebra we assembled some basic operations. The scalar, vector, and dyadic products are essential for general modeling tasks, and they are applied to specific geometric problems. Some of them we readied for the toolbox: straight line, plane, normal form of plane, plane projection tensor, and reflection tensor.

Needless to say, but worth emphasizing, we just got started! There is so much more for you in store. Although we already introduced frames and coordinate systems, we need to dig deeper. I shall attempt to clearly delineate their distinctly separate purposes in the next chapter.